LOGANATHA NARAYANASAMY GOVT. COLLEGE (Autonomous), PONNERI – 601 204
APRIL - 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **17PGE4C** Title of the Paper : **Elective - Stochastic Process**

Date : **26.09.2020 FN** Time : **90 Mins** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. Define second-order process.
2. Define Gaussian processes.
3. Write down the classifications of one dimensional process.
4. Define Markov Chain and transition probability matrix.
5. Define class property.
6. Define ergodicity.
7. Explain any two postulates for Poisson Process.
8. Define purely random Poisson process.
9. Define probability of extinction.
10. Define Erlang-k process.
11. Define renewal process.
12. Write the integral equation of renewal theory.

PART – B

Answer Any **THREE** Questions from the following

13. Show that the process $X(t) = A\cos\omega t + B\sin\omega t$ is covariance stationery, where A, B are uncorrelated random variables each with mean 0 and variance 1 and ω is a positive constant.
14. If $\{Z_i, i = 1,2,3,...\}$ be a sequence of i.i.d random variables with $E\{Z_i\} = 1$ and $X_n = \prod_{i=1}^{n} Z_i$. Prove that $\{X_n,\ n \geq 1\}$ is a martingale.
15. Derive Chapman-Kolmogorov equation of the Transition probabilities.
16. Prove that $\{X_n, n \geq 0\}$ be a Markov chain having state space $S = \{1,2,3,4\}$ and transition matrix

$$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$ is ergodic.

17. If $N(t)$ is a Poisson process then prove that the auto-correlation coefficient between $N(t)$ and $N(t+s)$ is $\{t/(t+s)\}^{1/2}$.
18. Derive Erlang's formula.
19. State and prove Renewal theorem.

PART – C

Answer Any **TWO** Questions from the following

20. Show that the process $\{X(t), t \epsilon T\}$ is non-stationary, whose probability distribution is given by $P\{X(t) = n\} = \begin{cases} \frac{(at).^{n-1}}{(1+at).^{n+1}} & n = 1,2,3,\ldots \\ \frac{(at)}{(1+at)}, & n = 0 \end{cases}$

21. Prove that the state j is persistent if and only if $\sum_{n=0}^{\infty} p_{ij}^{(n)} = \infty$

22. (i) If the intervals between successive occurrences of an event E are independently distributed with common exponential distribution with mean 1/λ, show that the events E

from the Poisson process with mean 1/λ.

(ii) Show that difference of two independent Poisson processes is not a poison process.

23. Discuss Erlang distribution.

24. If $p_n \to \alpha$, then prove that $v_n \to \alpha b$, where $b = \sum b_n$. If $\sum p_n$ converges to $\beta(<\infty)$ then $\sum v_n \to b\beta$.

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APRIL - 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **17PGE4D** Title of the Paper : **Elective - Fuzzy Sets and its applications**

Date : **28.09.2020 FN** Time : **90 Mins** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. What is a dominance relation? Check the dominance relation for u = (7,3,0,5) , v = (2,2,0,4) and $w=(3,4,1,4)$.
2. Let E be a set and $M=[0,1]$ its associated membership set. Find (i) $\bar{\underset{\sim}{A}}$ and (ii) $\bar{\underset{\sim}{A}}\cap\underset{\sim}{B}$ if $\underset{\sim}{A}=\{(x_1|0,2),(x_2|0,7),(x_3|1),(x_4|0),(x_5|0,5)\}$ and $\underset{\sim}{B}=\{(x_1|0,5),(x_2|0,3),(x_3|1),(x_4|0,1),(x_5|0,5)\}$.
3. Define a Fuzzy graph.
4. Define the complement of a relation and hence find the same for

$\underset{\sim}{\mathcal{R}}$	y_1	y_2	y_3	y_4
x_1	0,3	0,4	0,2	0
x_2	0,5	0	1	0,9
x_3	0,4	0	0,1	0,8

5. When do you call a relation to be antisymmetric?
6. Define a similitude subrelation.
7. If $\mathscr{D}_{\underset{\sim}{a}}=[0,6\ ,\ 0,9[$ and $\mathscr{D}_{\underset{\sim}{b}}=[0,2\ ,\ 0,7[$, then find the value of (i) $\mathscr{D}_{\underset{\sim}{a}\wedge\underset{\sim}{b}}$ and (ii) $\mathscr{D}_{\underset{\sim}{a}\vee\underset{\sim}{b}}$
8. What is a maximal monomial?
9. State the Law of external composition.
10. Define a fuzzy submonoid.
11. If $E=\{x_1,\ x_2\}$ and $M=\left\{0,\ \frac{1}{2},\ 1\right\}$, then find the power set of E.
12. Show that $\underset{\sim}{a}\wedge(\underset{\sim}{b}\vee\underset{\sim}{c})=(\underset{\sim}{a}\wedge\underset{\sim}{b})\vee(\underset{\sim}{a}\wedge\underset{\sim}{c})$.

PART – B

Answer Any **THREE** Questions from the following

13. Find the value of (i) $d(\underset{\sim}{A},\underset{\sim}{B})$ (ii) $\delta(\underset{\sim}{A},\underset{\sim}{B})$ (iii) $e^2(\underset{\sim}{A},\underset{\sim}{B})$ (iv) $\varepsilon(\underset{\sim}{A},\underset{\sim}{B})$ for

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$\underset{\sim}{A}=$	0,7	0,2	0	0,6	0,5	1	0

and

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$\underset{\sim}{B}=$	0,2	0	0	0,6	0,8	0,4	1

14. Find the first, second and global projection for the relation given below:

$\underset{\sim}{\mathcal{R}}$	y_1	y_2	y_3	y_4
x_1	0,1	0,2	1	0,3
x_2	0,6	0,8	0	0,1
x_3	0	1	0,3	0,6
x_4	0,8	0,1	1	0
x_5	0,9	0,7	0	0,5
x_6	0,9	0	0,3	0,7

15. (a) State and prove the theorem of decomposition for a similitude relation

(b) If $\tilde{\mathcal{R}}$ is transitive and reflexive, then prove that $\tilde{\mathcal{R}}^k = \tilde{\mathcal{R}}$, $k = 1, 2, 3, \ldots$

16. Determine the domain of $\tilde{f}(\tilde{a}, \tilde{b}) = \bar{\tilde{a}} \wedge \tilde{b}$, if $\tilde{a} \in \{0,2 \ ; \ 0,3 \ ; \ 0,4 \ ; \ 0,5\}$ and $\tilde{b} \in \{0 \ ; \ 0,1\} \cup \{0,7 \ ; \ 0,8 \ ; \ 0,9\}$

17. Construct a groupoid for the law $\tilde{A} * \tilde{B} = \tilde{A} \cap \tilde{B}$ with $E = \{A, B\}$ and $M = \left\{0, \frac{1}{2}, 1\right\}$.

18. Find $\tilde{B}$, if $\tilde{B} \subset E_2$ is conditioned on E_1 such that $E_1 = \{x_1, x_2, x_3\}$, $E_2 = \{y_1, y_2, y_3, y_4, y_5\}$, $\tilde{A} = \{(x_1|0,3), (x_2|0,7), (x_3|1)\}$ and

$\tilde{\mathcal{R}}$	y_1	y_2	y_3	y_4	y_5
x_1	0,8	1	0	0,3	0,7
x_2	0,8	0,3	0,8	0,4	0,7
x_3	0,2	0,3	0	0,2	1

19. Write short notes on the Gaussian fuzzy integers.

PART – C

Answer Any **TWO** Questions from the following

20. If $p_i, m_i, n_i \in \square^+$, $i = 1, 2, \ldots, k$; then prove that

$$\left(p_i \le m_i + n_i, \ i = 1, 2, \ldots, k\right) \Rightarrow \sqrt{\sum_{i=1}^{k} p_i^2} \ \le \ \sqrt{\sum_{i=1}^{k} m_i^2} + \sqrt{\sum_{i=1}^{k} n_i^2}.$$

21. Find the Max-min composition of $\tilde{\mathcal{R}}_1$ and $\tilde{\mathcal{R}}_2$ for the following relations

$\tilde{\mathcal{R}}_1$	y_1	y_2	y_3	y_4	y_5
x_1	0,1	0,2	0	1	0,7
x_2	0,3	0,5	0	0,2	1
x_3	0,8	0	1	0,4	0,3

$\tilde{\mathcal{R}}_2$	z_1	z_2	z_3	z_4
y_1	0,9	0	0,3	0,4
y_2	0,2	1	0,8	0
y_3	0,8	0	0,7	1
y_4	0,4	0,2	0,3	0
y_5	0	1	0	0,8

22. If $\hat{\tilde{\mathcal{R}}}$ is the max-min transitive closure of the resemblance relation $\tilde{\mathcal{R}}$, then prove that $\bar{\hat{\tilde{\mathcal{R}}}}$ is the min-max transitive closure of the corresponding dissemblance relation.

23. If $\tilde{f}(\tilde{a}, \tilde{b}, \tilde{c}) = (\bar{\tilde{a}} \wedge \bar{\tilde{b}}) \vee (\tilde{a} \wedge \tilde{b} \wedge \bar{\tilde{c}})$, then solve by the method of marinos that under what condition does the function of fuzzy variable satisfies $\alpha_{k-1} \le \tilde{f}(\tilde{a}, \tilde{b}, \tilde{c}) < \alpha_k$?

24. Explain the concept of Geometric fuzzy integers in detail and hence show that it forms a commutative monoid.

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APRIL - 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **17PGM4I** Title of the Paper : **Functional Analysis**

Date : **23.09.2020 FN** Time : **90 Mins** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. State the Hahn- Banach theorem
2. Define isomorphic isomorphism
3. State the Uniform Boundedness theorem
4. When do call a normed linear space N is called reflexive.
5. Define Hilbet space and write its properties
6. If H contains only the zero vector then whether it has orthonormal set t.
7. Define the self adjoint, unitary and normal operators
8. When do you say two projections P and Q are orthogonal?
9. Write the standard kronecker delta
10. When do we say two matrices are similar
11. Define orthonormal set in a Hilbert space H
12. Define Banach space

PART – B

Answer Any **THREE** Questions from the following

13. Let N and N' be normed linear spaces and T is a linear transformation of N to N'. Prove that the following are equivalent:

    a. T is continuous

    b. T is continuous at the origin, in the sense that $x_n \to 0 \Rightarrow T(x_n) \to 0$

    c. There is a real number $K \geq 0$ with the property that $\|T(x)\| \leq K\|x\|$ for every $x \in$

    d. If S is a closed unit sphere in N, then its image T(S0 is a bounded set in N'.

14. If N is a normed linear space, then prove that the closed unit sphere S* in N* is a compact Hausdorff space in the weak* topology.
15. Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
16. Prove that the adjoint operator $T \to T^*$ on $\mathcal{B}(H)$ has the following properties

    a) $(T_1 + T_2)^* = T_1^* + T_2^*$

    b) $(\alpha T)^* = \bar{\alpha}\, T^*$

    c) $(T_1 T_2)^* \quad T_2^* T_1^*$

    d) $T^{**} = T$

    e) $\|T^*\| = \|T\|$

    f) $\|T^* T\| = \|T\|^2$.

17. I f N_1 and N_2 are normal operators on H with the property that either commutes wit the adjoint of the other then prove that $N_1 + N_2$ and $N_1 N_2$ are normal .
18. State and prove closed graph theorem
19. If T is normal then prove that M_i's are pariwise orthogonal.

PART – C

Answer Any **TWO** Questions from the following

20. Let M be a linear subspace of a normed linear space N and let "f" be a functional defined on M. If x_0 is a vector not in M and if $M_0 = M + [x_0]$ is the linear subspace spanned by M and x_0 , then f can be extended to a functional on M_0 such that $\|f_0\| = \|f\|$.
21. State and prove open mapping theorem.
22. If M is a closed linear subspace of a Hilbert space H, then prove that $M \oplus M^{\perp}$ is also closed and $H = M \oplus M^{\perp}$
23. If P is a projection on H with range M and null space N then prove hat $M \perp N$ iff P is self-adjoint and in this case $N = M^{\perp}$
24. State and prove the spectral theorem.

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APRIL - 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **17PGM4J** Title of the Paper : **Calculus of Variations and Integral Equations**

Date : **24.09.2020 FN** Time : **90 Mins** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. Define a linear function.
2. When we say that a functional $v[y(x)]$ reaches a maximum on a curve $y = y_0(x)$.
3. Write the Euler equation.
4. Write the necessary conditions for an extremum in the problem with fixed boundary conditions.
5. State reflection-of-extremals problem.
6. State reciprocity principle.
7. Define control function.
8. Write Fredholm integral equations of second kind.
9. State Fredholm alternative theorem.
10. Define resolvent kernel.
11. State Fredholm's first theorem.
12. State Parseval's identity.

PART – B

Answer Any **THREE** Questions from the following

13. Discuss the problem of brachistochrone.
14. Discuss Refraction of extremals.
15. Find the broken-line extremals of the functional $v = \int_{x_o}^{x_1} y'^2(1-y')^2\,dx$.
16. Using the Ostrogradsky-Hamilton principle, find the equations of motion of a system of particles of mass $m_i\,(i = 1,2,...,n)$ with coordinates $\left(x_{i,} y_{i,} z_i\right)$ acted upon by forces having the force function $-U$, given the constraints $\varphi_j\left(t, x_1, x_2, ..., x_n, y_1, y_2, ...y_n, z_1, z_2, ..., z_n\right)$.
17. Find the resolvent kernel for the integral equation $g(s) = f(s) + \lambda \int_0^{2\pi} \left(st + s^2t^2\right) g(t)dt$.
18. Solve the symmetric integral equation,
    $g(s) = (s+1)^2 + \lambda \int_0^1 (st + s^2t^2) g(t)dt.$
19. Solve the integral equation $s = \int_0^s \frac{g(t)}{(s-t)^{1/2}}$.

PART – C

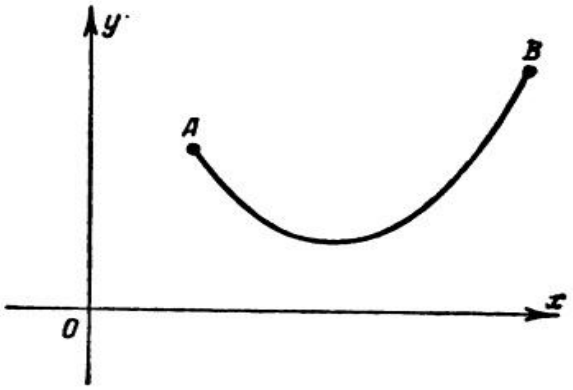
Answer Any **TWO** Questions from the following

20. State and prove the fundamental lemma of the calculus of variations.

21. Test for an extremum the function $v=\int_0^{x_1}\left(y'^2+z'^2+2yz\right)dx$, given that,

$y(0)=0;\quad z(0)=0$, and the point $\left(x_1,y_1,z_1\right)$ can move over the plane $x=x_1$.

22. Find the form of an absolutely flexible, nonextensible homogeneous rope of length l suspended at the points A and B (see the figure).



23. Solve the integral equation $g(s)=f(s)=\lambda\int_0^1(1-3st)g(t)dt$.

24. State and Prove Hilbert-Schmidt theorem.

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APRIL - 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **17PGM4K** Title of the Paper : **Fluid Dynamics**

Date : **25.09.2020 FN** Time : **90 Mins** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. Define stream lines.
2. Define velocity potential.
3. Define equipotential surfaces.
4. What is meant by Pitot tube?
5. What is a simple source?
6. Write Bernoulli's equation.
7. What is meant by two dimensional flow?
8. What is laminar flow?
9. Define complex velocity potential.
10. Write the Cauchy Riemann equations.
11. Write the six components of rate of strain.
12. What are different Types of Vortices?

PART – B

Answer Any **THREE** Questions from the following

13. Derive the equation of Continuity.
14. Prove that the pressure p at a point P in a moving inviscid fluid is same in all direction. .
15. Find the stoke's stream function for a uniform stream.
16. Discuss the uniform flow past an infinitely long circular cylinder.
17. Discuss the translational motion of fluid element.
18. At the point in an incompressible fluid having spherical polar co-ordinates (r, θ, ψ) the velocity components are $(2Mr^{-3}\cos\theta, Mr^{-3}\sin\theta, 0)$ where M is a constant. Show that velocity is of potential kind. Find the velocity potential and the equations of streamlines.
19. Discuss the flow due to a uniform line doublet at origin of strength □ □ per unit length and its axis being along the x-axis..

PART – C

Answer Any **TWO** Questions from the following

20. Test whether the motion given by $q = \frac{k^2(xj - yi)}{x^2 + y^2}$ is a possible motion of incompressible fluid. If so determine the equation of streamline.
21. Derive Bernoulli's equation of motion.
22. Find the stoke's stream function for a doublet at uniform line source along OZ.
23. A vortex circulation $2\pi R$ is at rest at the point $z = na (n>1)$ in the presence of a plane circular boundary $|z|=a$ around which there is a circulation $2\pi\lambda k$. Show that $\lambda = \frac{1}{n^2 - 1}$.
24. State and prove the Milne - Thomson circular theorem.

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APRIL – 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **PGE4A** Title of the paper : **Elective: Fuzzy sets and its Applications**

Date: **28.09.2020 FN** Time : **90 Mins.** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. If $\underset{\sim}{A}=\{(x_1|0,2),(x_2|0,7),(x_3|1),(x_4|0),(x_5|0,5)\}, \underset{\sim}{B}=\{(x_1|0,5),(x_2|0,3),(x_3|1),(x_4|0,1),(x_5|0,5)\}$ find $\underset{\sim}{A}\cap\overline{\underset{\sim}{B}}$.

2. Let E = $\{x_1, x_2\}$ and M = {0, 0.5, 1} Write the set of fuzzy subsets.

3. Prove that the transitive closure of any fuzzy binary relation is a transitive binary relation.

4. For the following data, find the first projection, second projection and global projection.

$\angle\underset{\sim}{\Re}$	Y_1	Y_2	Y_3	Y_4
X_1	0,1	0,2	1	0,3
X_2	0,6	0,8	0	0,1
X_3	0	1	0,3	0,6
X_4	0,8	0,1	1	0
X_5	0,9	0,7	0	0,5
X_6	0,9	0	0,3	0,7

5. Define resemblance relation.

6. Define anti symmetric fuzzy binary relation.

7. Let $\underset{\sim}{\wp}_a=[0,6\ ,0,9[$ *and* $\underset{\sim}{\wp}_b[0,2\ ,\ 0,7[$ .Find $\underset{\sim}{\wp}_{\underline{a}\wedge\underline{b}}$ *and* $\underset{\sim}{\wp}_{\underline{a}\vee\underline{b}}$

8. Write the difference between the probabilityconcept for fuzzy subsets and for ordinary subsets.

9. Define a fuzzy Groupoid .

10. Define fuzzy submonoid.

11. Define Dissemblance relation.

12. Define reducible fuzzy preorder.

PART – B

Answer Any **THREE** Questions from the following

13. Consider the fuzzy subset $\underset{\sim}{A}$: $\mu_{\underline{A}}(x_1)=0,7;\mu_{\underline{A}}(x_2)=0,9;\mu_{\underline{A}}(x_3)=0;\mu_{\underline{A}}(x_4)=0,6;\mu_{\underline{A}}(x_5)=0,5;\mu_{\underline{A}}(x_6)=1$ and

$$\pi_{\underline{A}}(x_i)=\frac{\mu_{\underline{A}}(x_i)}{\sum_{i=1}^{6}\mu_{\underline{A}}(x_i)}$$ Find $H(\pi_1,\pi_2,...,\pi_6)$.

14. Let $\underset{\sim}{\Re}$ be any fuzzy binary relation. If, for some k, $\underset{\sim}{\Re}^{k+1}=\underset{\sim}{\Re}^k$ then prove that $\hat{\underset{\sim}{\Re}}=\underset{\sim}{\Re}\cup\underset{\sim}{\Re}^2\cup...\cup\underset{\sim}{\Re}^k$.

15. The fuzzy binary relation $\underset{\sim}{\Re}$ is similitude then prove that $\overline{\underset{\sim}{\Re}}$ is dissimilitude relation.

16. Express the function $\underset{\sim}{f}(\underset{\sim}{a}.\underset{\sim}{b})=(\underset{\sim}{a}\wedge\overline{\underset{\sim}{a}})\vee(\overline{\underset{\sim}{a}}\wedge\underset{\sim}{b}\wedge\hat{\underset{\sim}{b}})$ explicitly in table of values.

17. Let $a=\mu_{\underset{\sim}{A}}(x), b=\mu_{\underset{\sim}{B}}(x), c=\mu_{\underset{\sim}{C}}(x)$ .Define $a\square\ b=a+kb-ab,\ k\in[0,1]$. Find k if associative law is true.

18. If $a=\mu_{\underset{\sim}{A}}(x), b=\mu_{\underset{\sim}{B}}(x), c=\mu_{\underset{\sim}{C}}(x)$ then check whether the following is true or not.

$(i)\overline{\underset{\sim}{A}\cdot\underset{\sim}{B}}=\overline{\underset{\sim}{A}}\ \hat{+}\ \overline{\underset{\sim}{B}}$ $(ii)\ \underset{\sim}{A}\cdot(\underset{\sim}{B}\hat{+}\underset{\sim}{C})=(\underset{\sim}{A}\cdot\underset{\sim}{B})\hat{+}(\underset{\sim}{A}\cdot\underset{\sim}{C})$.

19. Let $0\le\underset{\sim}{a}<\underset{\sim}{b}\le 1$ verify De Morgan's Theorem.

PART – C

Answer Any **TWO** Questions from the following

20. Let $p_i, m_i, n_i\in R^+$ I = 1,2,…,k then prove that $(p_i\le m_i+n_i,\ i=1,2,...,k)\ \Rightarrow\sqrt{\sum_{i=1}^{k}p_i^2}\ \le\sqrt{\sum_{i=1}^{k}m_i^2}\ +\sqrt{\sum_{i=1}^{k}n_i^2}$.

21. Prove that the following fuzzy binary relation is transitive.

$\underset{\sim}{\Re}$	A	B	C	D
A	0,2	1	0,4	0,4
B	0	0,6	0,3	0
C	0	1	0,3	0
D	0,1	1	1	0,1

22. Let $\underset{\sim}{\Re}\subset$ E ×E be a similitude relation. Let x,y,z be three elements of E and let $c=\mu_{\underset{\sim}{\Re}}(x,z)=\mu_{\underset{\sim}{\Re}}(z,x); a=\mu_{\underset{\sim}{\Re}}(x,y)=\mu_{\underset{\sim}{\Re}}(y,x); b=\mu_{\underset{\sim}{\Re}}(y,z)=\mu_{\underset{\sim}{\Re}}(z,y)$.Prove that c ≥ a = b or a ≥ b = c or b ≥ c = a.

23. Let $\underset{\sim}{f}(\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c})=(\underset{\sim}{a}\wedge\overline{\underset{\sim}{b}})\vee(\overline{\underset{\sim}{a}}\wedge\underset{\sim}{c})\vee\overline{\underset{\sim}{c}}$ and suppose that [0,1] is divided into three intervals [0,0,2[, [0,2 , 0,3[, [0,3 ,1]What conditions will give $\underset{\sim}{f}(\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c})\in I_k$.

24. Give the table representing the fuzzy groupoid such that E = { A, B} , M = { 0, $\frac{1}{2}$, 1} and $\mu_{\underset{\sim}{A}\cap\underset{\sim}{B}}(x)=\mu_{\underset{\sim}{A}}(x)\wedge\mu_{\underset{\sim}{B}}(x)$.

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APRIL – 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **PGE4C** Title of the paper : **Elective: Fuzzy sets and its Applications**

Date: **28.09.2020 FN** Time : **90 Mins.** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. If $\underset{\sim}{A}=\{(x_1|0,2),(x_2|0,7),(x_3|1),(x_4|0),(x_5|0,5)\}, \underset{\sim}{B}=\{(x_1|0,5),(x_2|0,3),(x_3|1),(x_4|0,1),(x_5|0,5)\}$ find $\underset{\sim}{A}\cap\overline{\underset{\sim}{B}}$.

2. Let E = $\{x_1, x_2\}$ and M = {0, 0.5, 1} Write the set of fuzzy subsets.

3. Prove that the transitive closure of any fuzzy binary relation is a transitive binary relation.

4. For the following data, find the first projection, second projection and global projection.

$\angle\underset{\sim}{\Re}$	Y_1	Y_2	Y_3	Y_4
X_1	0,1	0,2	1	0,3
X_2	0,6	0,8	0	0,1
X_3	0	1	0,3	0,6
X_4	0,8	0,1	1	0
X_5	0,9	0,7	0	0,5
X_6	0,9	0	0,3	0,7

5. Define resemblance relation.

6. Define anti symmetric fuzzy binary relation.

7. Let $\underset{\sim}{\wp}_a=[0,6\ ,0,9[$ *and* $\underset{\sim}{\wp}_b[0,2\ ,\ 0,7[$ .Find $\underset{\sim}{\wp}_{\underline{a}\wedge\underline{b}}$ *and* $\underset{\sim}{\wp}_{\underline{a}\vee\underline{b}}$

8. Write the difference between the probabilityconcept for fuzzy subsets and for ordinary subsets.

9. Define a fuzzy Groupoid .

10. Define fuzzy submonoid.

11. Define Dissemblance relation.

12. Define reducible fuzzy preorder.

PART – B

Answer Any **THREE** Questions from the following

13. Consider the fuzzy subset $\underset{\sim}{A}$: $\mu_{\underline{A}}(x_1)=0,7;\mu_{\underline{A}}(x_2)=0,9;\mu_{\underline{A}}(x_3)=0;\mu_{\underline{A}}(x_4)=0,6;\mu_{\underline{A}}(x_5)=0,5;\mu_{\underline{A}}(x_6)=1$ and

$$\pi_{\underline{A}}(x_i)=\frac{\mu_{\underline{A}}(x_i)}{\sum_{i=1}^{6}\mu_{\underline{A}}(x_i)}$$ Find $H(\pi_1,\pi_2,...,\pi_6)$.

14. Let $\underset{\sim}{\Re}$ be any fuzzy binary relation. If, for some k, $\underset{\sim}{\Re}^{k+1}=\underset{\sim}{\Re}^{k}$ then prove that $\hat{\underset{\sim}{\Re}}=\underset{\sim}{\Re}\cup\underset{\sim}{\Re}^2\cup...\cup\underset{\sim}{\Re}^{k}$.

15. The fuzzy binary relation $\underset{\sim}{\Re}$ is similitude then prove that $\overline{\underset{\sim}{\Re}}$ is dissimilitude relation.

16. Express the function $\underset{\sim}{f}(\underset{\sim}{a}.\underset{\sim}{b})=(\underset{\sim}{a}\wedge\overline{\underset{\sim}{a}})\vee(\overline{\underset{\sim}{a}}\wedge\underset{\sim}{b}\wedge\hat{\underset{\sim}{b}})$ explicitly in table of values.

17. Let $a=\mu_{\underset{\sim}{A}}(x), b=\mu_{\underset{\sim}{B}}(x), c=\mu_{\underset{\sim}{C}}(x)$ .Define $a\square\ b=a+kb-ab,\ k\in[0,1]$. Find k if associative law is true.

18. If $a=\mu_{\underset{\sim}{A}}(x), b=\mu_{\underset{\sim}{B}}(x), c=\mu_{\underset{\sim}{C}}(x)$ then check whether the following is true or not.

$(i)\overline{\underset{\sim}{A}\cdot\underset{\sim}{B}}=\overline{\underset{\sim}{A}}\ \hat{+}\ \overline{\underset{\sim}{B}}$ $(ii)\ \underset{\sim}{A}\cdot(\underset{\sim}{B}\hat{+}\underset{\sim}{C})=(\underset{\sim}{A}\cdot\underset{\sim}{B})\hat{+}(\underset{\sim}{A}\cdot\underset{\sim}{C})$.

19. Let $0\le\underset{\sim}{a}<\underset{\sim}{b}\le 1$ verify De Morgan's Theorem.

PART – C

Answer Any **TWO** Questions from the following

20. Let $p_i, m_i, n_i\in R^+$ I = 1,2,…,k then prove that $(p_i\le m_i+n_i,\ i=1,2,...,k)\ \Rightarrow\sqrt{\sum_{i=1}^{k}p_i^2}\ \le\sqrt{\sum_{i=1}^{k}m_i^2}\ +\sqrt{\sum_{i=1}^{k}n_i^2}$.

21. Prove that the following fuzzy binary relation is transitive.

$\underset{\sim}{\Re}$	A	B	C	D
A	0,2	1	0,4	0,4
B	0	0,6	0,3	0
C	0	1	0,3	0
D	0,1	1	1	0,1

22. Let $\underset{\sim}{\Re}\subset$ E ×E be a similitude relation. Let x,y,z be three elements of E and let $c=\mu_{\underset{\sim}{\Re}}(x,z)=\mu_{\underset{\sim}{\Re}}(z,x); a=\mu_{\underset{\sim}{\Re}}(x,y)=\mu_{\underset{\sim}{\Re}}(y,x); b=\mu_{\underset{\sim}{\Re}}(y,z)=\mu_{\underset{\sim}{\Re}}(z,y)$.Prove that c ≥ a = b or a ≥ b = c or b ≥ c = a.

23. Let $\underset{\sim}{f}(\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c})=(\underset{\sim}{a}\wedge\overline{\underset{\sim}{b}})\vee(\overline{\underset{\sim}{a}}\wedge\underset{\sim}{c})\vee\overline{\underset{\sim}{c}}$ and suppose that [0,1] is divided into three intervals [0,0,2[, [0,2 , 0,3[, [0,3 ,1]What conditions will give $\underset{\sim}{f}(\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c})\in I_k$.

24. Give the table representing the fuzzy groupoid such that E = { A, B} , M = { 0, $\frac{1}{2}$, 1} and $\mu_{\underset{\sim}{A}\cap\underset{\sim}{B}}(x)=\mu_{\underset{\sim}{A}}(x)\wedge\mu_{\underset{\sim}{B}}(x)$.

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APRIL - 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **PGE4D** Title of the Paper : **Elective - Financial Mathematics**

Date : **25.09.2020 FN** Time : **90 Mins** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. At what rate of simple interest Rs 1200 accumulate interest of Rs 72 in six months .
2. Find the discount value of Rs 1000 due in 3 months if the rate is 11%
3. Define accumulation.
4. Accumulate Rs 10000 at $j_{12} = 6\%$ for 5 years.
5. Define ordinary annuity and annuity due
6. A person deposits Rs 500 every 3 months into a saving account that pays interest at $j_4 = 4\%$. He made the first deposit on March 1, 2006. How much money will he have in his account after he make his deposit on September 1 ,2010?
7. Raja deposits Rs 100 at the beginning of each month for 3 years into an account paying $j_{12} = 4.5\%$. how much is in his account at the end of 3 years?
8. Define balloon payment.
9. On a roll two fair dices. Determine the probability that (a) a total of 7 (b) a total of 6 or 7 .
10. Define Mathematical expectation of a random variable X.
11. Write the formula for accumulation at the simple interest rate r.
12. Charlie pays Rs 1 to enter a betting game. If he can get 3 heads in a row by flipping a fair coin.

PART – B

Answer Any **THREE** Questions from the following

13. On January 10 Mr. Ram barrows Rs 1000 on a demand loan from his bank. Interest is paid at the end of each quarter and at the time of last payment. Interest is calculated at the rate of 12% on the balance of the loan outstanding. Mr. Ram repaid the loan with the following Payments
    March 1 Rs100
    April 17 Rs 300
    July 12 Rs 200
    August 20 Rs 100
    October 18 Rs 300
    Calculate the interest payments required and the total interest paid.
14. Gustav opens a saving account by depositing Rs 5000 on May 17. The account pays interest at 4 % per annum. How much is in the account on December 31 if a) the interest is compounded daily b) the interest is calculated daily and paid into the account at the end of each month.
15. A car selling for Rs 22500 may be purchased by paying Rs 2500 down and the balance in equal monthly payments for 5 years. Determine these monthly payments at (a) $j_{12}=12\%$ (b) $j_{12}=4.9\%$.
    Compare the result.
16. Determine the approximate interest rate j_4 at which Rs 275 each at the end of every 3 months will accumulate to Rs 5000 in 4 years?
17. In a dice game, one wins a number of Rupees equal to the number showing on the die's upward face, What the mathematical expectation of any participant?

18. Mr. X owes Rs 500 due in 4 months and Rs 700 due in 9 months. What single payment (a) now (b) in 6 months (c) in a year will liquidate these obligation if money is worth 11 % .
19. What is the equal payment for 10 years will pay off a loan of Rs 20000 with a interest at 10% compounded daily , if the first payment is made 6 months from now

PART – C

Answer Any **TWO** Questions from the following

20. Mr Hill has two options available in repaying a loan: He can repay Rs200 at the end of 5 months and Rs 300 at the end of 10 months or he can pay Rs X at the end of 3 months and Rs 2X at the end of 6 months. If the options are equivalent and money is worth 12%. Find X using as the focal date (a) the end of 6 months (b) end of 3 months.

21. How long will take Rs 500 to accumulate to Rs 840 at J_{12} = 123%. Assume that a) the exact method of accumulation is in effect
    b) the practical method of accumulation is in effect.

22. A person buys some equipment and signs a contract that calls for a down payment of Rs 1500 and for the payment of Rs 200 a month for 10 years. The interest rate is 12% compounded monthly
    a) What is the cash value of the contract?
    b)If he missed the first 8 payments of Rs 200 what must he pay at the time of ninth payment is due to bring himself up to date?
    c) If he missed the first 8 payments of Rs 200 what must he pay at the time of ninth payment is due to discharge indebtedness completely
    d) if at the beginning of the 5^{th} year the contract is sold to a buyer at a price that will yield j_{12}=15% what does the buyer pay?

23. A couple wants to accumulate Rs 10000 by making payments of Rs.800 at end of each half-year into an investment account that earns interest 1t J_2 = 9%. Calculate the number of full payments required and the size of the concluding payment using both the procedures
    Procedure 1= 969.43 Procedure-2 = 272.94.

24. You invest Rs 10000 for 1 year with ABC company which promises to make two payments of Rs 5569.28, the first 6 month and the second at the end of 1 year. If ABC is paying interest at j_2 = 15% what is the fair price you should pay for these two payments if a) the paymens are guaranteed b) the probability of default for the first payment is 2% and the probability that the first payment is made, but the second payment i not is 5%?

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APRIL - 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **PGM4C** Title of the Paper : **Calculus of Variations and Integral Equations**

Date : **24.09.2020 FN** Time : **90 Mins.** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. Define a linear functional.
2. Write the Euler equation.
3. Define the density of the Lagrangian function.
4. State transversality condition.
5. State the reflection-extremals problems.
6. Define holonomic constraints.
7. What do you mean by isoperimetric problems?
8. Define Degenerate Kernel.
9. State Fredholm Alternativetheorem.
10. Define complex symmetric of kernel.
11. State Fredholm's first theorem.
12. Write Cauchy principal integral.

PART – B

Answer Any **THREE** Questions from the following

13. Find the differential equations of the lines of propagation of light in an optically non-homogeneous medium in which the speed of light is $v(x, y, z)$.
14. On what curves $v[y(x)] = \int_0^{\frac{\pi}{2}} [y'^2 - y^2]dx, \quad y(0) = 0, \quad y\left(\frac{\pi}{2}\right) = 1.$
15. Find the transversality condition for functional of the form

    $v = \int_{x_0}^{x_1} A(x, y, z)\sqrt{1 + y'^2 + z'^2}\, dx$, if $z_1 = \varphi(x_1, y_1)$.
16. Find the external distance between two surfaces $z = \varphi(x, y)$ and $z = \psi(x, y)$.
17. Find the form of an absolutely flexible, non-extensible homogeneous rope of length *l* suspended at the points A and B.
18. Solve the integral equation$g(s) = f(s) + \lambda \int_0^1 (1 - 3st)g(t)dt$.
19. Evaluate the resolvent for the integral equation,

    $$g(s) = f(s) + \lambda \int_0^1 (s + t)g(t)dt.$$

PART – C

Answer Any **TWO** Questions from the following

20. State and prove the fundamental lemma of the calculus of variations.

21. Test for an extremum the function $\int_0^{x_1} \frac{\sqrt{1+y'^2}}{y} dx$, given that $y(0)=0,\ y_1 = x_1 - 5$.

22. Find the extremals of the isoperimetric problem:

$v[y(x)] = \int_0^1 \left(y'^2 + x^2\right) dx$ given that $\int_0^1 y^2 dx = 2;\ \ y(0) = 0; y(1) = 0.$

23. Find the eigen values and eigen functions of the homogeneous integral equation,

$g(s) = \lambda \int_1^2 \left(st + \left({}^1/_{st}\right)\right) g(t)dt.$

24. Solve the symmetric Fredholm integral equation of the first kind,

$\int_0^t K(s,t)g(t)dt = f(s)$ where $K(s,t) = \begin{cases} s(1-t),\ s < t. \\ (1-s)t,\ s > t. \end{cases}$

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APRIL - 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **PGM4E** Title of the Paper : **Functional Analysis**

Date : **23.09.2020 FN** Time : **90 Mins** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. Define second conjugate space of N.
2. Define normed linear space.
3. State the Open Mapping theorem.
4. Define Hilbert space.
5. Define self-adjoint operator.
6. Prove the adjoint operation $T \to T^*$ on (H) has $\| T^* \| = \| T \|$.
7. Define Banach subalgebra.
8. Define spectrum of T.
9. Define Banach* algebra.
10. Define a multiplicative functional.
11. When do you say the normed linear space N is said to be isometrically isomorphic to N'.
12. State Bessel's Inequality.

PART – B

Answer Any **THREE** Questions from the following

13. Let N and N' be normed linear spaces and T a linear transformation of N into N'. Then prove the following conditions on T are all equivalent to one another:
    (1) T is continuous.
    (2) T is continuous at the origin, in the sense that $x_n \to 0 \Rightarrow T(x_n) \to 0$.
14. State and prove Schwarz inequality.
15. If T is an operator on H for which $(Tx, x) = 0$ for all x, then $T = 0$.
16. Prove that the set of all topological divisors of zero is a subset of the set of singular elements.
17. If f_1 and f_2 are multiplicative functional on A with the same null space$M$, prove $f_1 = f_2$.
18. Let M be a closed linear subspace of a Hilbert space H, let x be a vector not in M, and let d be the distance from x to M. Prove that there exists a unique vector y_0 in M such that $\| x - y_0 \| = d$.
19. If r is an element of R, then prove $1 - r$ is regular.

PART – C

Answer Any **TWO** Questions from the following

20. State and prove the Hahn-Banach Theorem.
21. State and prove the Uniform Boundedness Theorem.
22. Prove that A closed linear subspace M of H reduces an operator T $\Leftrightarrow$ M is invariant under both T andT^*.
23. Prove that $r(x) = \lim \| x^n \|^{1/n}$.
24. State and prove the Gelfand-Neumark theorem.

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APRIL - 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **PGM4F** Title of the Paper : **Stochastic Process**

Date : **26.09.2020 FN** Time : **90 Mins** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. Define state space. When it is continuous?
2. When do you say Gaussian processes is strictly stationary
3. Define order of Markov chain
4. Define one step transition probability.
5. A radioactive source emits particle at a rate of per minute in accordance with a Poisson process. Each particle emitted has a probability 0.6 of being recorded. In a 4 minute interval what is the probability that the number of particles recorded is 10
6. write the postulates of Poisson process
7. Define renewal process
8. Define pure birth process
9. State Blackwell's theorem
10. Define periodic and aperiodic
11. Define martingales process
12. When do you say the processes are of independent increment?

PART – B

Answer Any **THREE** Questions from the following

13. Classify the random process
14. Discuss the random walk with absorbing barriers at 0 and 4. Whenever it is at any position r ($0<r<4$) it moves to r+1 with probability p or to r-1 with probability q , p+q = 1 . but as soon as it reaches 0 or 4 it remains there itself. Find the transition probability matrix for the above
15. Prove that "Sum of two independent poisson process is a poisson process."
16. Suppose that a certain system can be considered to be in two states " opening" and under repair denoted by 1 and 0 respectively. Suppose that of operating period and period under repair are independent random variable having negative exponential distribution with mean 1/b and 1/a respectively (a,b >0) . Find the Kolmogrorov forward equation.
17. Let $X = (X_t)_{t\_0}$ be a renewal process with mean inter-arrival time μ and renewal function $m(t) = E[X_t]$. Then $m(t)/t \rightarrow 1/\mu$ as $t\rightarrow\infty$
18. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute then in an arrival of 2 minutes the probability that the number of customers arriving is (i)exactly 4 (ii) greater than 4 (iii) less than 4
19. Show that $x(t) = A_1 + A_2t$ is evolutionary where A_1 and A_2 are independent r.v's with $E(A_i) = a_i$ and $var(A_i) = \sigma_i^2$ i=1,2

PART – C

Answer Any **TWO** Questions from the following

20. Show that the process $X(t)$ whose probability distribution undercertain conditions is given by

P(x(t)=n)=$\begin{cases} \frac{(at)^{n-1}}{(1+at)^{n-1}}, n = 1,2, \dots \\ \frac{at}{1+at}, n = 0 \end{cases}$ is evolutionary .

21. Classify the states for the following tpm $\begin{pmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}$.
22. Let N{t) denote the number of occurrences of aspecified event in an interval of length t. Using the postulates of Poisson Process derive the Poisson law
23. Derive Chapman- Kolmogorov equation
24. Prove that "the renewal function M satisfies the equation M(t) = F(t) $+\int_0^t M(t-x)dF(x)$".

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APRIL - 2020 SEMESTER EXAMINATIONS
IV SEMESTER – M.Sc., MATHEMATICS

Paper Code : **PGM4G** Title of the Paper : **Calculus of Variations and Integral Equations**

Date : **24.09.2020 FN** Time : **90 Mins** Maximum Mark:**75 Marks**

PART – A

Answer Any **FIVE** Questions from the following

1. Define k^{th} order proximity of curves.
2. Write the Ostrograsdy equation.
3. State the law of reflection of light.
4. Write the transversality condition for $v[y(x)] = \int_{x_0}^{x_1} F(x, y, z, y', z')\, dx$.
5. State reciprocity principle.
6. State isoperimetric problem.
7. Define normalized function.
8. State Neumann series.
9. State Fredholm's first theorem.
10. Write Plemelj formula.
11. Define symmetric kernel.
12. Write the Euler-Poisson equation.

PART – B

Answer Any **THREE** Questions from the following

13. State and prove the Fundamental Lemma of Calculus of Variations.
14. Find the transversality condition for functional of the form,
    $v = \int_{x_0}^{x_1} A(x, y, z)\sqrt{1 + y'^2 + z'^2}\ dx$ if $z_1 = \varphi(x_1, y_1)$.
15. Obtain the shortest distance between two points $A(x_0, y_0, z_0)$ and B(x_1, y_1, z_1) on the surface $\emptyset(x, y, z) = 0$.
16. Invert the integral equation $g(s) = f(s) + \lambda \int_0^{2\pi} (sinscost) g(t) dt$.
17. Solve the integral equation $f(s) = \int_a^s \frac{g(t)dt}{[(\cos t - \cos s)^{1/2})]}, 0 \le a < s < b \le \pi$.
18. Find the extremal of the functional
    $v[y(x)] = \int_0^1 (1 + y''^2)\ dx\ , y(0) = 0,\ y'(0) = 1, y(1) = 1\ , y'(1) = 1$
19. Prove that the eigen functions of a symmetric kernel corresponding to different eigen values are orthogonal.

PART – C

Answer Any **TWO** Questions from the following

20. Obtain the Euler's equation for the functional $v[y(x), z(x)] = \int_{x_0}^{x_1} F(x, y, z, y', z')\, dx$.
21. Discuss the reflection-of-extremals problem.
22. Using the Ostrogradsky-Hamilton principle, find the equations of motion of a system of particles of mass m_i with usual coordinates acted upon by forces having the force function-U.
23. Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 [(s + t)]\ g(t) dt$ and find the eigen values.
24. Solve the integral equations $g(s) = 1 + \lambda \int_0^{\pi} [\sin\ (s + t)]\ g(t) dt$.

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